Modeling Vowel Sounds in a Male Singing Voice

Don Bosley
Master of Music Technology Candidate,
Music Technology Department
New York University
donbosleymusic@gmail.com

I. INTRODUCTION AND PURPOSE

The human voice is able to generate a wide variety of timbres through processes which come naturally, and therefore may appear simple on the surface. The reality is that there is a complex array of intertwining actions occurring in order to create these varied timbres. The source of the voice is air being forced from the lungs which travels over the vocal folds causing them to rapidly open and close. A series of modifiers, which include the larynx, lips and nostrils, collectively create the "vocal tract." **[1]** The jaw, tongue and lips are collectively known as "articulators." **[1]** All of these systems and their individual components interact in order to alter and shape the nature of the sound output from the mouth.

Composers and music technologists, as well as scientists interested in many forms of communication have either studied, or gained inspiration from the human voice. The general purpose has been to create a model for use in everything from synthesis applications to data reduction in telephone lines. The results vary from electro-mechanical emulators, Homer Dudley's "Voder", and in more recent times methods of analysis/re-synthesis **[2].** The purpose of this particular experiment is to gain a basic understanding of the relationship between the harmonic series contained in a few vowel sounds, and the role these harmonics play in shaping timbre. The sets of collected data will be analyzed and used in order to "build" an instrument which emulates a handful of these vocalizations and operate in real-time on a system that is not necessarily very robust.

II. THE VOCAL TRACT AS A TUBE

The vocal tract behaves in a similar manner to a tube connected to a "flow-controlled reed"; in general, tubes of this type have the ability to support odd-numbered modes**[1]**. As waveforms are analyzed in section IV of this paper, it will be important to keep in mind the odd-numbered modes and the similarities between vowel sounds and triangular waveforms, which are based on odd harmonics. The vocal tract, articulators, and the air source themselves all interact in order to shape the output of the sound.

If one performs a simple experiment and sings different vowels while paying detailed attention to the actions of the body a few things become apparent. If various vowels are sung and air is continuously pushed from the lungs as the transitions are made, one should notice that the greatest effect on the timbre results from slight variations in the position of the tongue, but more drastically the size and shape of the opening of the mouth. The mouth is changing the opening of a resonant tube, in this case the vocal tract, thus altering its standing wave modes. The result is modification of the apparent amplitudes of the resultant harmonics.

III. BREIF HISTORY OF VOCAL SYNTHESIS

Although many attempts have been made to mimic human speech through electromechanical means, a large portion of today's work in vocal synthesis is based on the principles outlined by Homer Dudley during his time at Bell Labs. Dudley's invention of the Voder and subsequent Vocoder both utilized a source-filter model where the signal was decomposed and re-synthesized using banks of filters excited by a pulse train or other wideband noise signal **[3]**.

Methods of additive, subtractive, frequency modulation, and physical modeling have all been utilized to synthesize vocal sounds and speech **[2]**. Source-filter models have been updated and expanded through the use of increased computational power available in modern machines. Various methods which also incorporate the simulation of vocal formants have become popular in research as well.

One early version of this formant synthesis was FOF (*function d'onde formantique* in French). FOF utilizes sets of parallel band-pass filters which are excited by a pulse train, or equivalent waveform. Each filter passes a sound at the respective frequency, and these frequencies are summed in order to give the overall spectrum of the desired sound **[2]**. FOF is advantageous in that individual parameters, such as envelopes and sideband content, can be controlled precisely within each grain. Methods resembling FOF will be focused on during experimentation however other forms of synthesis include Models of Resonance, VOSIM, window function synthesis and linear predictive coding **[2]**.

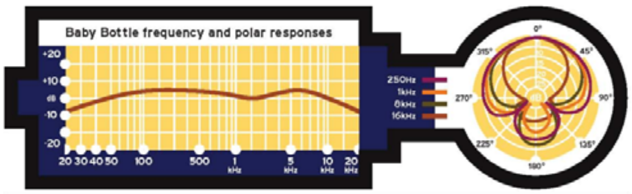
IV. INITIAL EXPERIMENTS

The entire basis of the initial experiments was to understand parallels between the singing voice and synthesis techniques the author had a general understanding of. The voice can essentially be described as a synthesizer contained within one's body. Sung vowels serve as an excellent example of the variety of the types of sounds that can be generated and for this reason they will be the focus of this section.

The first step was to record audio samples which were collected from a human subject. In this case, the subject was a male vocalist whose voice lies under the classification of 'bass' in the concert music world. The note 'G' was selected as the target and would be sung in three different octaves. The frequencies of the actual pitch, which were 97.997, 195.998, and 391.991 Hz, were each sung as three different vowels: "ooo" as in *boot*, "aah" as in *saw*, and "eee" as in *we*.

All nine samples were recorded at a sampling rate of 44.1 kHz at 16 bits using a Blue Baby Bottle® condenser microphone. The subject was projecting approximately 6-8 inches away from the diaphragm and a windscreen, also known as a pop filter was between the subject and the microphone. The microphone has a relatively, but not perfectly flat frequency response and the subject was placed at 0^{o}**[4]**. **(FIG. 1)**

FIG. 1

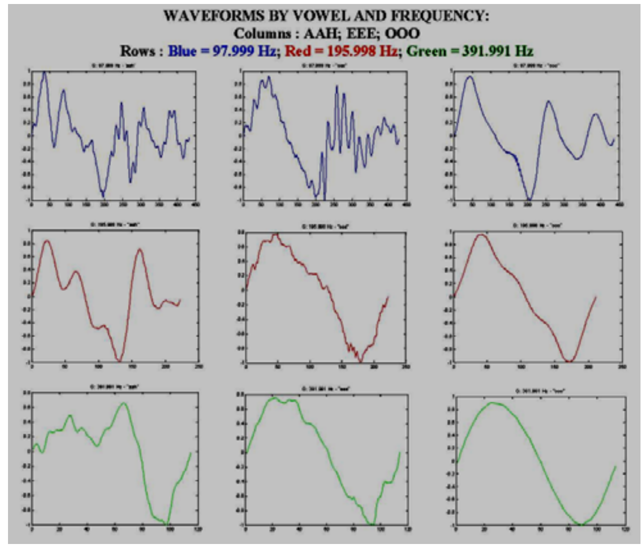


The subject was given instructions to match the pitch of sine tones which were delivered to him through a pair of headphones. The subject used little to no vibrato where possible and took extended breaths between the vocalization of the different vowels. The vowels were then sustained until the subject was out of breath and repeated until satisfactory samples of each vowel set were collected for each frequency.

Portions of the recording were then extracted and examined for sections containing steady state waveforms which wavered little in pitch or amplitude. The goal was to identify a few consecutive cycles of the waveform whose properties appeared as similar as possible. These consecutive cycles were then isolated and further separated into individual or single cycles. Although an attempt was made to average the properties of each of the waveforms into one composite, it was found to have a low-pass filtering (averaging) effect which seemed detrimental to preserving the true shape, and thus spectral content, of the waveform.

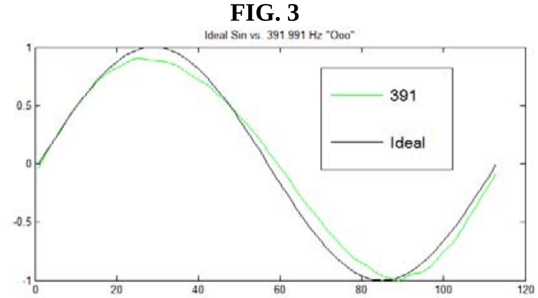
What was eventually decided upon was to extract a single cycle of the waveform **(FIG. 2)** and test them using a looping method in order to evaluate the waveforms natural qualities. By utilizing wavetable synthesis with the individual cycle, the comparisons to the natural vowel sound could be made in order to determine how ideal the waveforms were. Although the wavetable playback was devoid of the natural variation of a live human performance, the general quality of the vowel remained with each and selections were made.

FIG. 2



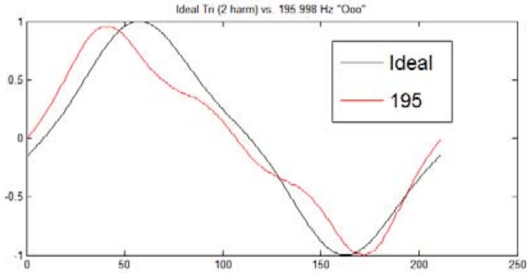
Without attempting to extract any spectral information, the author compared the resulting waveforms to those utilized in "classic" synthesis, such as triangle, sine, square, and/or sawtooth. As an example, the author quickly picked out the "ooo" waveform at 391 Hz as resembling a simple sine wave. Compare an idealized sine wave at 391.991 Hz to the sung waveform of similar frequency **(FIG. 3)** there are similarities noticeable by the eye alone.

FIG. 3



A similar event occurred when looking at the "ooo" vowel for 195.998 Hz. The author noticed a similarity between the classic triangle wave and this sample. Although the triangle wave would not contain many harmonics and still have the curved appearance that this waveform does, there were still some similarities. **(FIG. 4)** A triangle wave containing only two harmonics was generated and as with the previous example the contour, crest, trough, and overall shape of the waveforms are comparable.

FIG. 4

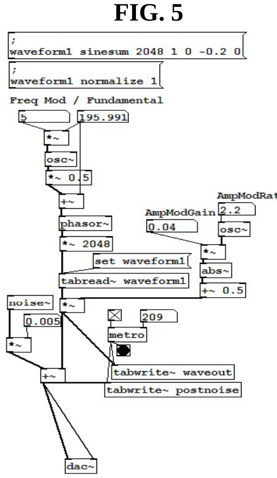


In order to re-synthesize the sound of the vowels, a number of simple methods were considered. The first approach was to try to generate the vowels utilizing additive synthesis. Since the 391 Hz “ooo” was so close to a sine wave, it seemed like a good starting point. A patch was generated in Miller Puckette’s open-source *Pure Data* (PD) in order to accomplish this.

Although the quality of the vowel sound was apparent there was something human lacking – the subtle variations natural to almost all acoustic sounds. Amplitude modulation (AM) was then applied to the end of the signal chain in order to vary the intensity of the sound. This helps to simulate the variation in pressure when the lungs are pushing air. Frequency modulation (FM) was also applied; in this case a randomized wobble (not a steady vibrato, which can be deliberately applied by trained vocalists) was used to simulate natural variations in pitch. This was more pleasing, but the model still lacked.

Because the “ooh” at 195 Hz was also close to an idealized classic waveform, additive synthesis was also used to reproduce it. Again, PD was the platform for generating the sound, and the techniques of amplitude and frequency modulation were both applied. **(FIG. 5)** Because multiple sine waves were added, an additional message box utilizing PD’s *Sinesum* function was implemented. *Sinesum* allows a list to be generated containing the amplitudes of individual harmonics in a single, easily manageable object.

FIG. 5



Attempting to bring the waveforms closer in shape, the amplitude of the second harmonic was adjusted away from the ideal. This yielded a positive, but still not quite satisfying result. A small amount of white noise was also injected into the signal in order to achieve additional continuously changing random variation in the values comprising the waveform.

Still dissatisfied with the overall results, another model was sought; it was decided that construction by source-filter modeling may yield more pleasing results. In this case the starting point was a source consisting of a glottal excitation waveform as described in multiple sources (Cook, Howard and Angus, Roads, et al…)**[5]**.**(FIG. 6)** Because of ease of use for real-time spectral visualization, Cycling ‘74’s MAX/MSP was utilized for this portion of the experiment. **(FIG. 7)**

FIG. 6

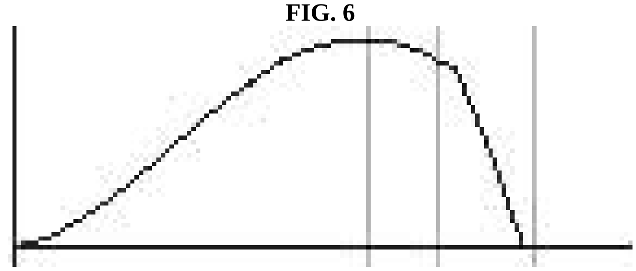
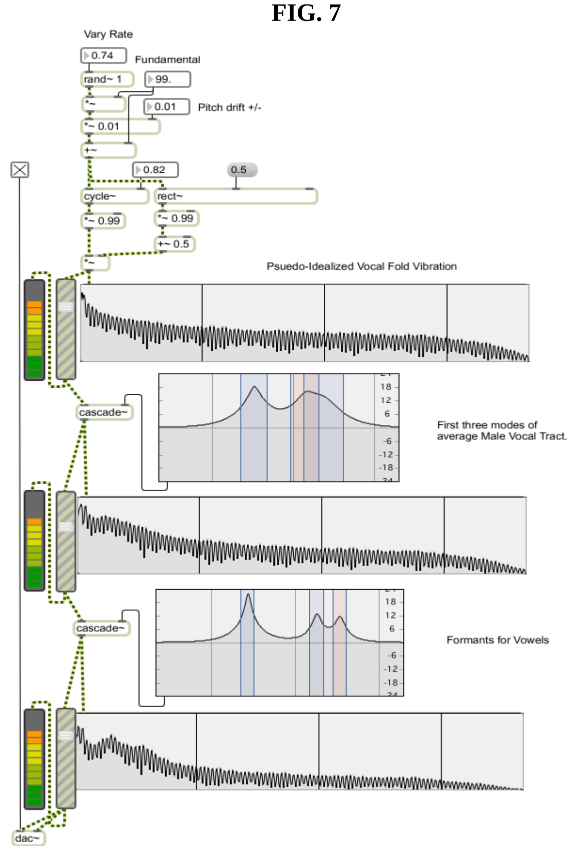
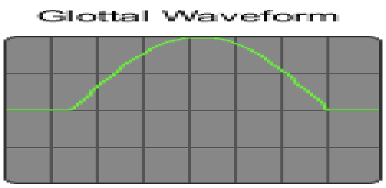


FIG. 7



The patch was generated using a half-sine, half square wave in order to approximate the waveform previously described for glottal excitation of the vocal folds. **(FIG. 8)** Although this waveform doesn't have the asymmetry, and therefore sidebands and additional frequency content as the one displayed in **(FIG. 7)**, it does serve to pulse other portions of the simulations fairly successfully. As with earlier patches created in PD, a variation in the frequency of the fundamental was created, however a random value generator was inserted in order to give the appearance of the variation a more natural, or less correlated, regular or cyclic appearance.

FIG. 8



The filtering portion happens in two steps. The first is a cascade of filters which model the average resonances contained within a male vocal tract based on the vowel. The second step filters the sound based on user input for average formants when vowels are spoken. Although the affect of spoken vowel formants are different than those of sung vowels, the data was available for the former **[1]**.

V. NOISE BASED FORMANT SYNTHESIZER

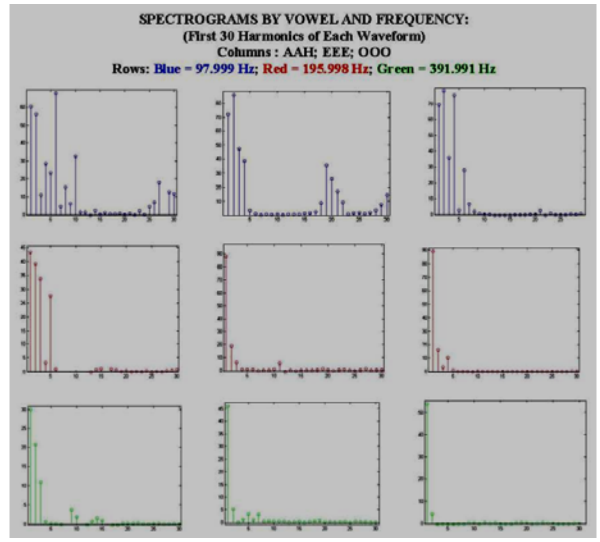
Reconsidering the type of sound quality being produced by previously attempted methods a formant based method with ideas drawn from historical work was attempted. Formants are essentially the standing wave modes contained within the vocal tract **[1]**. In terms of content and use within the instrument to be constructed, they will appear as peaks on a spectrogram. In order to try to accurately synthesize these sounds a bank capable of creating 30 harmonics will be utilized. At a sampling rate of 44.1 kHz this allows for fundamental frequencies of up to 735 Hz to be used without introducing aliasing into the signal.

In examining methods to increase the accuracy of the model, the precision of some factors were evaluated. The first change made was to use the frequency of the sampled waveforms rather than those of the target pitch. The actual frequencies were determined by calculating the sampling rate divided by the number of samples contained within each wavetable. **(FIG. 9)** As evidenced by the chart, there was some deviation and in terms of calculating the appropriate harmonic frequency values, this slight shift in the fundamental can cause detrimental effects in the values generated for the upper harmonics.

FIG. 9

	"OOO"		"EEE"		"AAA"	
Freq:	# Samples	Actual Freq	# Samples	Actual Freq	# Samples	Actual Freq
97	436	101.15	429	102.80	432	102.08
195	211	209.00	222	198.65	222	198.65
391	113	390.27	114	386.84	115	383.48

FIG. 10



A function was programmed in MatLab which accepted the correct fundamental frequencies from the chart above and used them to calculate accurate measurements of the first 30 expected harmonics for each signal. The values of the harmonics were then used to calculate the appropriate frequency bin from a 2,048 point FFT. **(FIG.10)** Because most of the answers were not whole numbers, for example results like bin 9.6 or 4.84, linear interpolation was used to weight the measurements of the two adjacent bins, giving a more accurate magnitude for the energy of each specific harmonic, rather than just produce an approximate measurement from a single bin.

In order to quickly test the accuracy of the magnitude information of the harmonics, data was once again placed into a PD object implementing the *Sinesum* function. **(FIG. 11)** Using a 4096 point wavetable, the harmonics were summed and normalized then the output waveform was graphically displayed and played back over loudspeakers. Not only did the waveforms have a similar visual character, they sounded very close to the source. As with the initial experiments involving methods of wavetable sampling, they lacked variation.

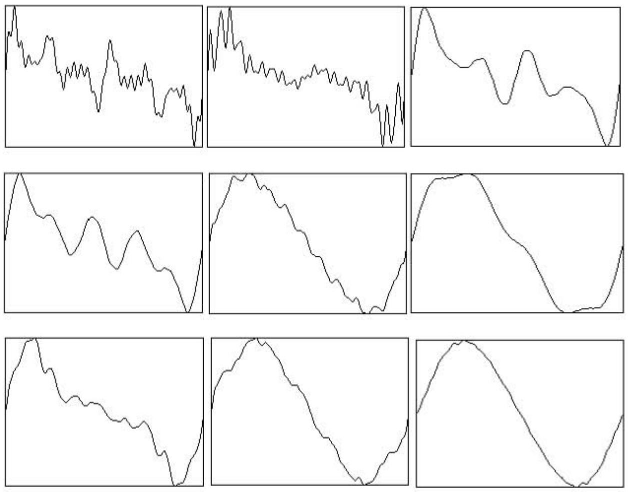
FIG. 11

```
;
waveform1 sinesum 4096 88.9785 15.9118 3.21184 10.1301
0.706506 -0.08946 -0.159651 -0.181032 -0.255613 0.066322
-0.0235096 0.0185279 -0.0205908 0.0399461 -0.0879088
-0.152437 -0.0159551 -0.0171355 -0.0747003 -0.0497116
-0.105362 -0.11497 -0.0324519 -0.119168 -0.0130321
-0.0338559 0.0110072 0.0411777 0.17697 0.200275;
waveform1 normalize
```

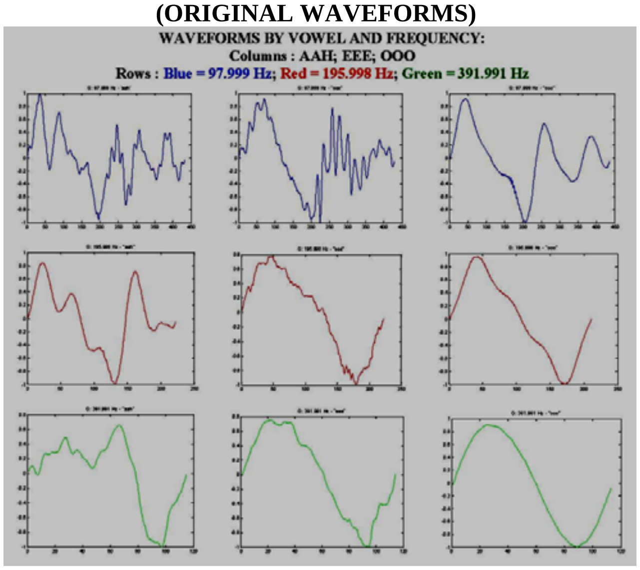
The results were fairly congruent with the appearance of the physical waveforms in terms of the nature and number of the harmonics contained in the signal. **(FIG. 12)** What is noticeable in both instances is how integral register is to timbre in the voice. In the lower ranges the waveforms are rich and contain a complex array of harmonic content. As the higher end of the range is approached the waveforms become simplified.

FIG. 12

PD SINESUM USING MAGNITUDES:
(First 30 Harmonics of Each Waveform)
Columns : AAH; EEE; OOO
Rows: 97.999 Hz; 195.998 Hz; 391.991 Hz



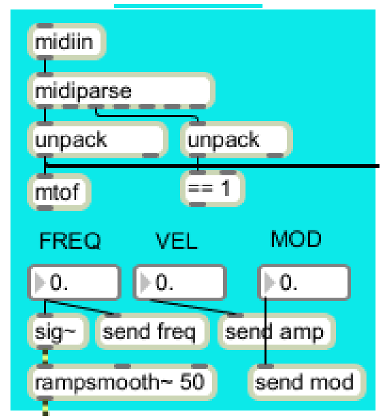
(ORIGINAL WAVEFORMS)



What is also noticeable is playing the waveforms back at significantly lower of higher pitches than the analysis was performed on yields unrealistic sounding results. For instance, the 97 Hz "aah" sounds too rich when played at 195, or 391 Hz. The 391 Hz "aah" sounds too thin when played at 195 or 97 Hz. Listening to these waveforms at alternate pitches proves the importance of register and its effect on timbre. In considering resonant modes of a tube, there is a natural series which generally loses power as the frequency increases. Although a strong fundamental may be produced in the upper register, its harmonic content could never be as strong as the lower register as a result of a fundamental limit of physics.

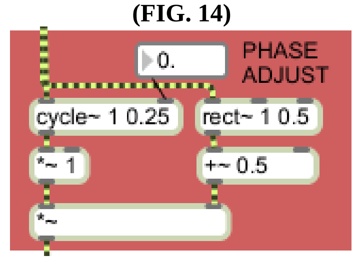
In order to emulate these waveforms, a subtractive synthesis method which attempts to isolate the formants of the voice was created in Max/MSP. The first step was establishing a real-time control input. Using a number of MIDI objects, control information could be input using an external MIDI keyboard. **(FIG. 13)**

(FIG. 13)



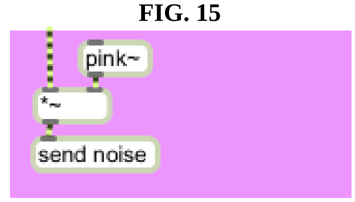
The first block of the synthesis engine was to generate the glottal excitation. **(FIG. 14)** This was again accomplished utilizing a half sine wave, half rectangle wave. **(see FIG. 8 for waveshape)** A phase adjust floating number box was also inserted because during initial stages it was found the phases of the waveforms would drift over time, changing the harmonic content of the waveform.

(FIG. 14)



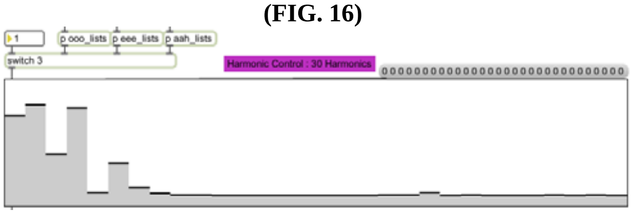
The next stage of the signal chain was to generate noise and excite it using the pulse created in the previous stage. **(FIG. 15)** Both white and pink noise were considered but because pink noise contains equal energy per octave, versus whites' equal energy across the entire spectrum, it was chosen**. [2]** Pink noise is often described as having less hiss or appears to have less high frequency content and was found to be easier and more appropriate for this application. Noise naturally provides the AM necessary for realistic function.

FIG. 15

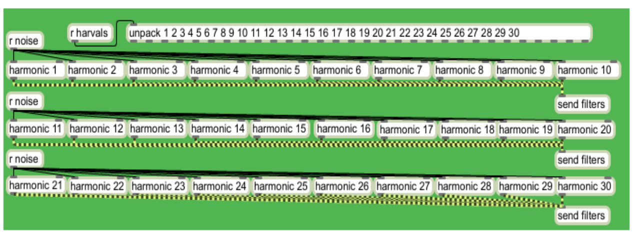


The data collected from the sampled waveforms was input into a selectable matrix which used a switch object to select numerically between vowels. The note numbers from the MIDI commands were then mapped to the appropriate spectral magnitudes for each register. **(see APPENDIX A for Magnitude values)** For instance when the G at 97 (or 98 Hz according to the Max MIDI-to-Frequency or *mtof* object) is played, the harmonics are scaled differently than when the G at 195 Hz is played. The information is then output to a graphic representation of the 30 harmonics **(FIG. 16)** which forwards the values to a series of 30 variably-tuned resonant bandpass filters. **(FIG. 17)**

(FIG. 16)

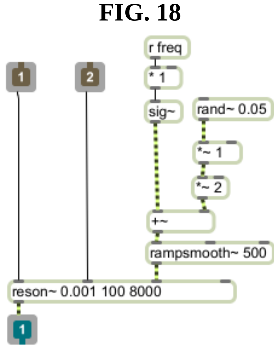


(FIG. 17)



Each filter in the bank of 30, acts as a gate for a particular formant to pass through. By setting the Q factor of each filter very high, only the specified frequency passes and others appear to not be present. This method resembles subtractive synthesis, carving away unnecessary frequency components and leaving spectral peaks at the necessary formants. Each harmonic is generated by a harmonic subpatch (in this case the first harmonic). **(FIG. 18)** Each subpatch includes its own randomized frequency modulation scaled by that harmonic's particular frequency and changes. The subpatch also defines initial conditions for resonant bandpass filter.

FIG. 18

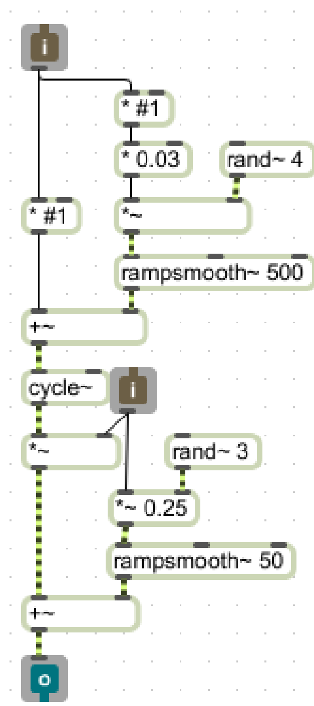


Although this model worked in real-time, there were a number of problems. In order to keep passbands tight, the Q factor had to be kept extremely high. The result of this is that resonances, particularly at low frequencies are too strong. This is a result of high feedback coefficients in the recursive portion of the filter. Resonance is very natural and having recursive structures allows for past events to determine the character of the present, but the required resonance in this case turned out to be detrimental to clarity in terms of identifying the quality of the vocal sounds. Many ooo and eee sounds end up sounding like aahs. An overall amplitude envelope was applied to increase the realism of the sound, but in terms of vocal synthesis this method was only partially musically useful.

VI. FUTURE WORK: SINE BANK SYNTHESIZER

In examining all of the methods utilized the PD *Sinesum* contained the most natural sounding synthesis. The next project will use what the author perceives as the best features of all of the work generated over the course of this paper. A bank of 30 individual sine wave generators containing frequency dependent AM and FM will be used. **(FIG. 19)**

FIG. 19



These will not be driven by a glottal pulse, since the magnitudes already account for the overall shape of the waveform. Randomized AM and FM will account for the variation that was provided by the noise in the Noise Based Formant Synthesizer. A series of interpolated values will be assigned for each note input so that a smooth variation occurs across registers rather than a hard break as transitions occur between octaves. **(APPENDIX B)** The modulation continuous controller will also be implemented to smoothly transition or blend between vowel sounds, having the value zero represent all ‘ooo’, 64 represent ‘eee’, and 127 represent ‘aah’.

VII. RESULTS AND VARIATIONS

Although the modifiers and articulators can be simulated through traditional audio filtering processes, a physical modeling approach which utilizes variables such as force and mass may be more accurate. Because all parts of the system are constantly in motion, the magnitude of portions of the spectrum are constantly in motion as well. A truly accurate simulation of the voice would account for these spectral changes at an almost microscopic level. Since the ultimate source of the voice is the vibration of the vocal fold itself, the flow of air may be the variable that requires the most focus. Certainly with modification of vowel sounds, the manner in which the mouth controls the shape of the vocal tract is also paramount.

Overall, the models constructed during experimentation were not terribly far from the types of vowels created during singing. Simulation of variation, as stated above, is the key factor. However, the remaining elements of realism required would require additional time, experimentation, observation, and probing into details relating to the mechanics of the voice.

REFERENCES

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[2] Curtis Roads. The Computer Music Tutorial. Cambridge, MA : MIT Press, 2001.

[3] Youngmoo Edmund Kim. Singing Voice Analysis/Synthesis. September, 2003.

[4] Blue Baby Bottle Owner’s Manual. Westlake Village, CA : Blue Microphones, 2007.

[5] Perry R. Cook . Doctoral Dissertation. September, 1991.

Appendix A: Non-Normalized Magnitudes of Harmonics in Sampled Waveforms

Harmonic #	G97_eee	G97_ooo	G97_aah	G195_eee	G195_ooo	G195_aah	G391_eee	G391_ooo	G391_aah
1	72.08	69.39	60.26	87.83	88.98	43.45	45.99	53.91	30.00
2	85.91	78.10	55.98	18.81	15.91	39.29	5.15	4.09	20.77
3	47.38	35.96	10.97	6.27	3.21	33.63	(0.06)	(0.38)	10.76
4	38.84	75.22	28.61	0.85	10.13	3.11	1.09	(0.26)	0.64
5	3.32	2.66	23.41	0.78	0.71	27.68	3.18	(0.40)	0.13
6	1.42	27.89	67.98	0.89	(0.09)	0.92	1.04	(0.34)	0.03
7	0.34	6.43	4.47	(0.10)	(0.16)	(0.58)	3.03	(0.29)	(0.08)
8	0.76	1.86	15.36	0.41	(0.18)	(0.83)	0.37	(0.16)	(0.35)
9	0.64	0.42	6.07	0.28	(0.26)	(0.37)	0.48	0.09	3.61
10	0.73	0.00	32.80	1.04	0.07	(0.46)	0.40	0.08	1.80
11	0.24	(0.11)	1.45	5.71	(0.02)	(0.44)	0.41	(0.25)	(0.60)
12	0.82	(0.54)	1.51	(1.00)	0.02	(0.36)	0.42	0.26	(0.20)
13	0.76	(0.44)	(0.34)	0.20	(0.02)	(0.17)	(0.08)	(0.09)	0.71
14	0.83	(0.42)	2.32	(0.19)	0.04	0.66	0.08	0.15	1.44
15	1.64	(0.48)	0.22	0.14	(0.09)	1.10	0.48	(0.24)	0.85
16	1.88	(0.31)	1.30	0.37	(0.15)	(0.47)	0.05	(0.25)	(0.48)
17	2.56	(0.22)	0.33	0.36	(0.02)	0.84	0.23	(0.16)	(0.33)
18	8.89	(0.17)	0.46	0.59	(0.02)	0.44	0.54	0.14	(0.32)
19	35.63	0.04	0.87	1.39	(0.07)	0.04	0.83	0.19	0.18
20	25.89	(0.03)	(0.08)	0.50	(0.05)	0.29	0.15	0.03	0.13
21	17.06	2.20	0.77	0.06	(0.11)	(0.09)	0.08	(0.15)	0.13
22	9.25	(0.55)	(0.26)	0.66	(0.11)	0.07	0.01	(0.11)	0.29
23	0.92	0.60	2.06	0.84	(0.03)	0.03	0.14	0.19	(0.02)
24	1.79	(0.25)	0.09	0.27	(0.12)	0.26	0.29	(0.07)	0.02
25	1.97	(0.24)	4.51	0.08	(0.01)	(0.19)	(0.17)	(0.14)	0.09
26	1.29	(0.36)	6.90	0.45	(0.03)	0.14	0.36	(0.17)	0.13
27	2.06	0.30	18.01	1.33	0.01	(0.04)	0.28	(0.17)	(0.02)
28	3.61	(0.09)	(2.70)	0.21	0.04	0.22	0.08	(0.16)	0.01
29	7.79	0.46	12.60	0.56	0.18	0.38	0.02	(0.04)	0.11
30	14.61	(0.31)	11.71	0.47	0.20	0.60	0.02	(0.07)	(0.03)

APPENDIX B: Possible values for Smooth Transitions across Registers

G 97	G#	A	B	C	C#	D	D#	E	F	F#	G 195	G#	A	B	C	C#	D	D#	E	F	F#	G 391
69.39	71.84	74.29	74.29	76.73	79.18	79.18	81.63	84.08	84.08	86.53	88.98	84.60	80.21	80.21	75.83	71.45	71.45	67.06	62.68	62.68	58.30	53.91
78.10	70.33	62.56	62.56	54.78	47.01	47.01	39.23	31.46	31.46	23.69	15.91	14.43	12.96	12.96	11.48	10.00	10.00	8.52	7.05	7.05	5.57	4.09
35.96	31.86	27.77	27.77	23.68	19.58	19.58	15.49	11.40	11.40	7.30	3.21	2.76	2.31	2.31	1.87	1.42	1.42	0.97	0.52	0.52	0.07	(0.38)
75.22	67.09	58.95	58.95	50.81	42.68	42.68	34.54	26.40	26.40	18.27	10.13	8.83	7.53	7.53	6.24	4.94	4.94	3.64	2.34	2.34	1.04	(0.26)
2.66	2.41	2.17	2.17	1.92	1.68	1.68	1.44	1.19	1.19	0.95	0.71	0.57	0.43	0.43	0.29	0.15	0.15	0.01	(0.12)	(0.12)	(0.26)	(0.40)
27.89	24.39	20.90	20.90	17.40	13.90	13.90	10.40	6.91	6.91	3.41	(0.09)	(0.12)	(0.15)	(0.15)	(0.18)	(0.21)	(0.21)	(0.25)	(0.28)	(0.28)	(0.31)	(0.34)
6.43	5.61	4.78	4.78	3.96	3.14	3.14	2.31	1.49	1.49	0.66	(0.16)	(0.18)	(0.19)	(0.19)	(0.21)	(0.23)	(0.23)	(0.24)	(0.26)	(0.26)	(0.28)	(0.29)
1.86	1.60	1.35	1.35	1.09	0.84	0.84	0.58	0.33	0.33	0.07	(0.18)	(0.18)	(0.17)	(0.17)	(0.17)	(0.17)	(0.17)	(0.17)	(0.16)	(0.16)	(0.16)	(0.16)
0.42	0.33	0.25	0.25	0.17	0.08	0.08	(0.00)	(0.09)	(0.09)	(0.17)	(0.26)	(0.21)	(0.17)	(0.17)	(0.13)	(0.08)	(0.08)	(0.04)	0.00	0.00	0.05	0.09
0.00	0.01	0.02	0.02	0.03	0.03	0.03	0.04	0.05	0.05	0.06	0.07	0.07	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.08
(0.11)	(0.10)	(0.09)	(0.09)	(0.08)	(0.07)	(0.07)	(0.06)	(0.05)	(0.05)	(0.03)	(0.02)	(0.05)	(0.08)	(0.08)	(0.11)	(0.14)	(0.14)	(0.16)	(0.19)	(0.19)	(0.22)	(0.25)
(0.54)	(0.47)	(0.40)	(0.40)	(0.33)	(0.26)	(0.26)	(0.19)	(0.12)	(0.12)	(0.05)	0.02	0.05	0.08	0.08	0.11	0.14	0.14	0.17	0.20	0.20	0.23	0.26
(0.44)	(0.39)	(0.33)	(0.33)	(0.28)	(0.23)	(0.23)	(0.18)	(0.12)	(0.12)	(0.07)	(0.02)	(0.03)	(0.04)	(0.04)	(0.05)	(0.06)	(0.06)	(0.07)	(0.08)	(0.08)	(0.09)	(0.09)
(0.42)	(0.36)	(0.30)	(0.30)	(0.25)	(0.19)	(0.19)	(0.13)	(0.07)	(0.07)	(0.02)	0.04	0.05	0.07	0.07	0.08	0.09	0.09	0.11	0.12	0.12	0.13	0.15
(0.48)	(0.43)	(0.38)	(0.38)	(0.33)	(0.28)	(0.28)	(0.23)	(0.18)	(0.18)	(0.14)	(0.09)	(0.11)	(0.13)	(0.13)	(0.15)	(0.17)	(0.17)	(0.19)	(0.20)	(0.20)	(0.22)	(0.24)
(0.31)	(0.29)	(0.27)	(0.27)	(0.25)	(0.23)	(0.23)	(0.21)	(0.19)	(0.19)	(0.17)	(0.15)	(0.16)	(0.18)	(0.18)	(0.19)	(0.20)	(0.20)	(0.21)	(0.22)	(0.22)	(0.23)	(0.25)
(0.22)	(0.19)	(0.17)	(0.17)	(0.14)	(0.12)	(0.12)	(0.09)	(0.07)	(0.07)	(0.04)	(0.02)	(0.03)	(0.05)	(0.05)	(0.07)	(0.09)	(0.09)	(0.10)	(0.12)	(0.12)	(0.14)	(0.16)
(0.17)	(0.15)	(0.13)	(0.13)	(0.12)	(0.10)	(0.10)	(0.08)	(0.06)	(0.06)	(0.04)	(0.02)	0.00	0.02	0.02	0.04	0.06	0.06	0.08	0.10	0.10	0.12	0.14
0.04	0.02	0.01	0.01	(0.00)	(0.02)	(0.02)	(0.03)	(0.05)	(0.05)	(0.06)	(0.07)	(0.04)	(0.01)	(0.01)	0.02	0.06	0.06	0.09	0.12	0.12	0.15	0.19
(0.03)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.04)	(0.03)	(0.03)	(0.02)	(0.01)	(0.01)	(0.00)	0.01	0.01	0.02	0.03
2.20	1.91	1.62	1.62	1.33	1.05	1.05	0.76	0.47	0.47	0.18	(0.11)	(0.11)	(0.12)	(0.12)	(0.12)	(0.13)	(0.13)	(0.13)	(0.14)	(0.14)	(0.14)	(0.15)
(0.55)	(0.49)	(0.44)	(0.44)	(0.38)	(0.33)	(0.33)	(0.28)	(0.22)	(0.22)	(0.17)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)
0.60	0.52	0.44	0.44	0.36	0.28	0.28	0.20	0.13	0.13	0.05	(0.03)	(0.00)	0.02	0.02	0.05	0.08	0.08	0.11	0.13	0.13	0.16	0.19
(0.25)	(0.23)	(0.22)	(0.22)	(0.20)	(0.18)	(0.18)	(0.17)	(0.15)	(0.15)	(0.14)	(0.12)	(0.11)	(0.11)	(0.11)	(0.10)	(0.10)	(0.10)	(0.09)	(0.08)	(0.08)	(0.08)	(0.07)
(0.24)	(0.21)	(0.18)	(0.18)	(0.16)	(0.13)	(0.13)	(0.10)	(0.07)	(0.07)	(0.04)	(0.01)	(0.03)	(0.04)	(0.04)	(0.06)	(0.08)	(0.08)	(0.09)	(0.11)	(0.11)	(0.12)	(0.14)
(0.36)	(0.32)	(0.28)	(0.28)	(0.23)	(0.19)	(0.19)	(0.15)	(0.11)	(0.11)	(0.07)	(0.03)	(0.05)	(0.07)	(0.07)	(0.09)	(0.10)	(0.10)	(0.12)	(0.14)	(0.14)	(0.16)	(0.17)
0.30	0.26	0.23	0.23	0.19	0.16	0.16	0.12	0.08	0.08	0.05	0.01	(0.01)	(0.03)	(0.03)	(0.06)	(0.08)	(0.08)	(0.10)	(0.13)	(0.13)	(0.15)	(0.17)
(0.09)	(0.07)	(0.05)	(0.05)	(0.04)	(0.02)	(0.02)	(0.01)	0.01	0.01	0.03	0.04	0.02	(0.01)	(0.01)	(0.04)	(0.06)	(0.06)	(0.09)	(0.11)	(0.11)	(0.14)	(0.16)
0.46	0.42	0.39	0.39	0.35	0.32	0.32	0.28	0.25	0.25	0.21	0.18	0.15	0.12	0.12	0.09	0.07	0.07	0.04	0.01	0.01	(0.02)	(0.04)
(0.31)	(0.24)	(0.18)	(0.18)	(0.12)	(0.05)	(0.05)	0.01	0.07	0.07	0.14	0.20	0.17	0.13	0.13	0.10	0.07	0.07	0.03	0.00	0.00	(0.03)	(0.07)